



Oxford Cambridge and RSA

Friday 20 June 2025 – Afternoon

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 (a) A student claims that the order of 5 modulo 37 is 5.
Without calculation, explain why this student's claim must be incorrect. [1]
- (b) Show that the order of 5 modulo 37 is 36. [3]
- 2 The points P , Q , R and S have coordinates $(1, 2, 4)$, $(3, 1, 6)$, $(4, 8, -2)$ and $(5, 5, 5)$ respectively.
- (a) Find, in exact form, the area of triangle PQR . [3]
- (b) Find, in exact form, the volume of tetrahedron $PQRS$. [3]
- (c) Use your answers to part (a) and part (b) to deduce the shortest distance from S to the plane containing P , Q and R . Give your answer in exact form.
[You may use the formula $V = \frac{1}{3}Ah$, where V is the volume of a pyramid, A is the area of one of its bases and h is the height relative to the base.] [2]
- 3 Let $N = 10a + b$ and $M = a - 3b$, where a and b are positive integers and $0 \leq b \leq 9$.
- (a) By considering $3N + M$, prove that $31 \mid N$ if and only if $31 \mid M$. [4]
- (b) Use a procedure based on the result of part (a) to show that $31 \mid 7\,167\,758$. [2]
- 4 In this question you must show detailed reasoning.
- The surface S is given by the equation $z = 4x^2y - 3xy^2 - 5x$ for all real values of x and y .
- (a) The point $P(1, 1, -4)$ lies on S .
Find the equation of the tangent plane to S at P . [4]
- (b) The point Q is a stationary point of S with positive x - and y -coordinates.
- (i) Find, in exact form, the coordinates of Q . [5]
- (ii) Find the value of the determinant of $\mathbf{H}$, the Hessian matrix of S , at Q . [4]
- (iii) Deduce the nature of the stationary point Q . [1]

- 5 (a) Determine the solution of the linear congruence $3x \equiv 7 \pmod{13}$. Give your answer in the form $x \equiv a \pmod{13}$, where $0 \leq a < 13$. [2]
- (b) Hence or otherwise, determine the solution of the simultaneous linear congruences $3x \equiv 7 \pmod{13}$, $7x \equiv 3 \pmod{31}$. [4]
- 6 In this question, the set S is $\{2, 4, 6, 8\}$, and $\times_{10}$ is the operation of multiplication modulo 10.
- (a) The group G consists of the set S , under the operation $\times_{10}$.
- (i) Complete the Cayley table for G given in the Printed Answer Booklet. [1]
- (ii) State the identity element of G . [1]
- (iii) State the order of each non-identity element of G . [2]
- (b) The group H consists of all 2×2 matrices of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, where $a, b \in S$, under the operation of matrix multiplication (using $\times_{10}$ to multiply elements of S).
- (i) State, with justification, the order of H . [2]
- (ii) State also the identity element of H . [1]
- (iii) State all the elements of H of order 2. [2]
- (iv) Using the result of part (a)(iii), or otherwise, explain why all the (non-identity) elements which do not have order 2 must have order 4. [3]
- (v) State whether or not H is cyclic. You must give a reason for your answer. [1]
- 7 In this question you must show detailed reasoning.
- A curve is defined parametrically by $x = \cos^3 t$, $y = \sin^3 t$, for $0 \leq t \leq \frac{1}{2}\pi$. When the curve is rotated through 2π about the x -axis, a surface of revolution is formed with surface area A .
- Determine the exact value of A . [8]

- 8 (a) The members of the family of the sequences $\{G_n\}$ satisfy the **first-order** recurrence relation $G_{n+1} = 1.25 - qG_n$, for $n \geq 0$, where q is a positive constant.
- (i) Determine, in terms of q , the general solution of the first-order recurrence relation. [3]
- (ii) Hence explain why, when q is close to zero, G_n is approximately constant and equal to 1.25. [2]
- (iii) In the case where $G_1 = 1.22$ and $G_2 = 1.2378$, find the values of q and G_0 . [3]
- (b) The population of a species of fish F_n is being modelled by the **second-order** recurrence relation

$$F_{n+2} = 1.25F_{n+1} - rF_{n+1}F_n,$$

where r is a positive constant and n is the number of weeks since the start of the breeding season.

You are given that the value of r is such that the model predicts that the population approaches a limiting value, L .

- (i) Determine L in terms of r . [2]
- (ii) Determine what happens if a term of the sequence is greater than $\frac{5}{4r}$. [2]
- (iii) Explain why the modified recurrence relation $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ may be preferable when modelling this fish population. [1]
- (iv) When $r = 0.0001$, both $F_{n+2} = 1.25F_{n+1} - rF_{n+1}F_n$ and $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ predict that the fish population approaches a limiting value.

By finding the limiting value for each model, when $F_0 = 2450$ and $F_1 = 2475$, compare the two models for the fish population. [3]

END OF QUESTION PAPER

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